The Design of a 13.625 GHz Structure Used for the Transformer Ratio Enhancement Experiments

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Introduction

In general, the wakefield theorem [1] restricts the maximum accelerating field behind the drive bunch in a wakefield accelerator to be less than twice the maximum retarding field inside the drive bunch thus limiting the efficiency which can be obtained. One of the concepts central to the physics of wakefield acceleration is the transformer ratio, R, defined as R = (Maximum energy gain behind the bunch)/(Maximum energy loss inside the drive bunch). For the case of a collinear drive and witness beam geometry device, R is less than 2 except in a few special cases [2]. However, using a ramped bunch train (RBT) where a train of n electron drive bunches, with increasing ('ramping') charge, one can achieve $R = 2n$ after the bunch train. We believe this method is feasible from an engineering standpoint using existing technology and an experiment to be preformed at the Argonne Wakefield Accelerator (AWA) is planned.

We designed a 13.625 GHz disk-loaded structure that consists with 40 cells for this planned experiment.

The Design of a Transformer Ratio Enhancement Structure by SuperFish

The initial parameters are shown in Table 1.

Table 1 The initial parameters in calculation

Operating Frequency	13625 MHz
Wavelength λ_0	22.003 mm
Mode	$2\pi/3$ TW
Cavity Length d	7.334 mm
Outer Radius b	9.653 mm
Inner Radius a	5.000 mm
Thicker of Iris t	2.000 mm

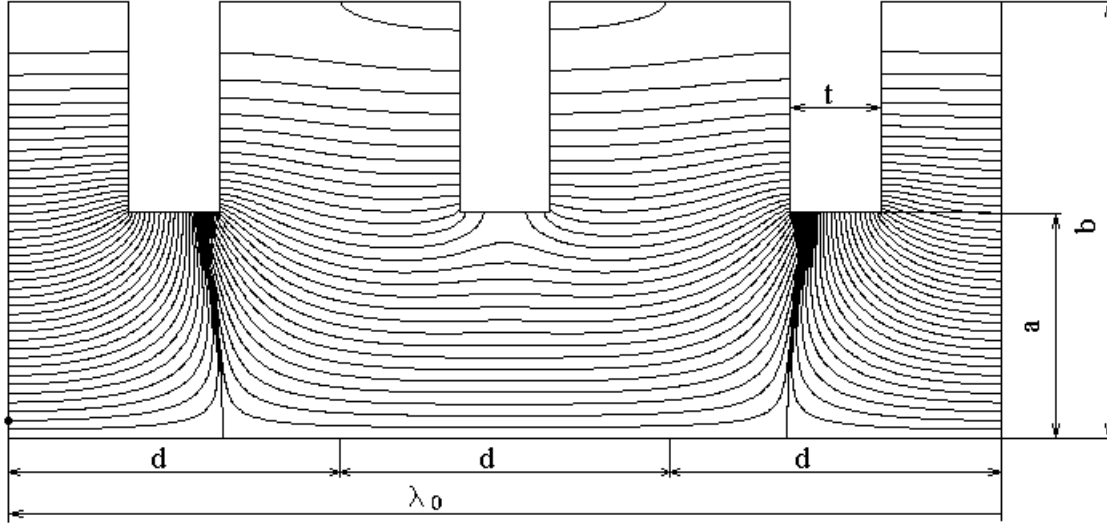The structure and field pattern is shown in Fig.1 which is calculated by SuperFish.

Fig.1 2π/3 mode disk-loaded structure designed by SuperFish for Ramped Bunch Train Experiment and the field pattern in it. The dimensions are shown in Table.1. The operating frequency is 13625.621 MHz.

Calculation of Wake Fields in a Transformer Ratio Enhancement Structure by Mafia

By using the geometry from SuperFish, we designed a 39 cavity plus 2 half-cavity structure for Ramped Bunch Train Experiment shown in Fig.2 and calculated the wakefield produced in this designed structure by one electron bunch of bunch length 4 mm and charge magnitude $Q = 1$ C, shown in Fig. 3.
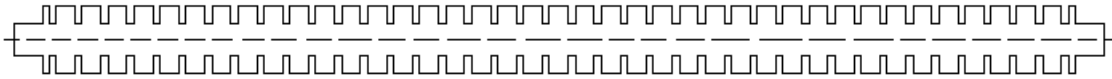

Fig.2 40 cavities Transformer Ratio Enhancement structure to be used for RBT. The wakefield will be calculated by using Mafia 2-D module under 300, 000 meshes.
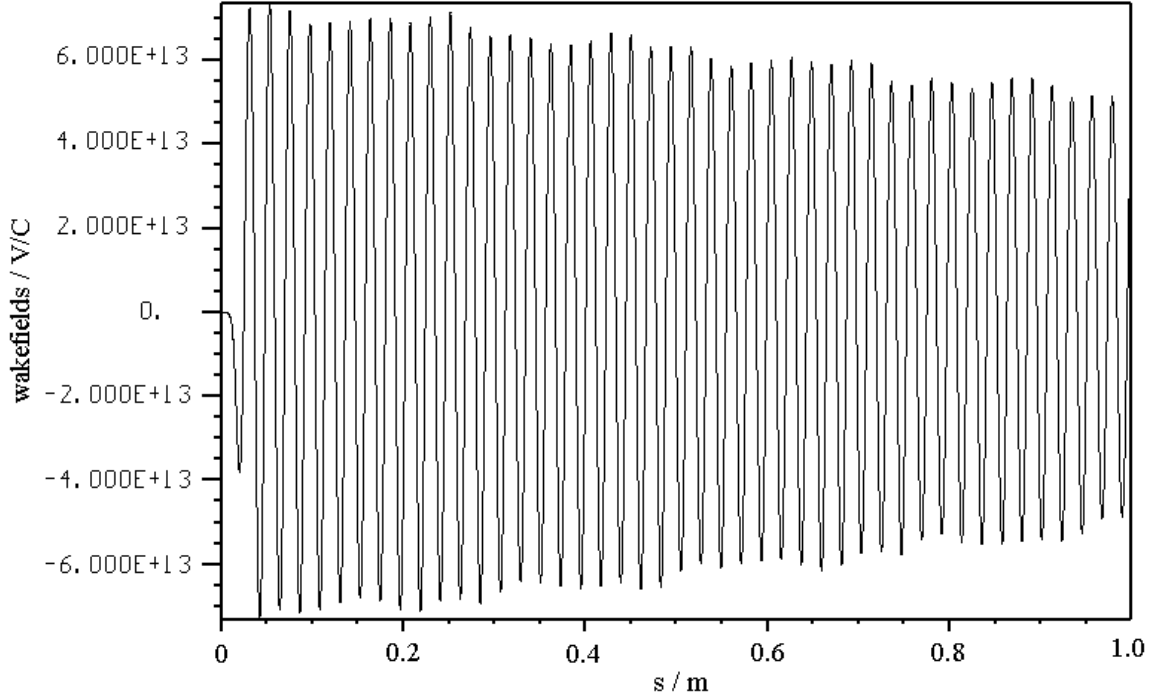
Fig.3 The wakefield produced in the designed structure by one electron bunch of bunch length 4 mm and charge magnitude Q = 1 C. The operating frequency is 13679.5 MHz.

Design of RBT at AWA Facility

According to the prediction in [2] and the results from SuperFish and Mafia, we plan a Ramped Bunch Train experiment which operating frequency is 13625 MHz.

From the RBT Algorithm for Transformer Ratio Enhancement [2], we have the following equations. Given the transformer ratio after the first *(n=0)* bunch *(R_0)* then the maximum transformer ratio that can be achieved after the *n th* bunch *(R_n)* is,

$R_n = (n+1)R_0$ *(n = 0,1, 2, ...N-1)*   (1)

This maximum enhancement of the transformer ratio can only be achieved if the following conditions are satisfied. The separation between the bunches is *1/2* integer or,

$d = (m + 1/2) \lambda_0$ *(m = 1, 2, ...,N-1)*   (2)

where λ_0 is the fundamental wavelength of the structure. The charge ratio of the individual bunches within the train increases according to,

$Q_n = Q_0 [nR_0 +1]$ *(n=0,1, 2, ...N-1)*   (3)

where Q_0 is the charge in the first drive bunch.

In addition to the requirements of spacing (Eqn. 2) and relative charge ratio (Eqn. 3) for the bunch train, there are also two requirements on the single bunch for obtaining maximal transformer ratio enhancement. The first condition is in addition to the work of [3]. The self-wake generated by the single bunch must be symmetric with respect to the center of the bunch. In other words, for a gaussian distribution, centered at $z = 0$, the self-wake within the bunch must satisfy, $W_0^-(-z) = W_0^-(+z)$. If this 'symmetric, single wake' condition is not met, the self-wakefields of the trailing bunch will be phase shifted from the

accelerating bucket of the leading bunch resulting in only a partial cancellation of the fields. This can be corrected by changing the spacing of the bunches within the train so that full cancellation is obtained, but since it is easier to generate a bunch train of equal spacing we choose to satisfy the above condition. Numerical simulations show that the 'symmetric, single wake' condition is met when the bunch length satisfies

$$s/?_0 = 0.2 \tag{4}$$

For a given R_0, the transformer ratio is maximized if the conditions specified in Eqn. 2, Eqn. 3, and Eqn. 4 are satisfied. This means that if $R_0 = 1$, then the fastest R_n can increase is $R_0 = 1$, $R_1 = 2$, $R_2 = 3$, $R_3 = 4$, etc. However, if $R_0 = 2$, then R_0 could increase as $R_0 = 2$, $R_1 = 4$, $R_2 = 6$, $R_3 = 8$, etc. Therefore, since our goal is to maximize R_n we desire $R_0 = 2$ and therefore, bunch length satisfies,

$$s/?_0 = 0.25 \tag{5}$$

For our experiment we use a bunch length of $s = 4$ mm. The wavelength $l_0 = 22.003$ mm. Then we have $s/l_0 = 0.18$, which is near the RBT algorithm requirement $s/l_0 = 0.2$. Then we can meet the 'symmetric, single wake' condition and make the Transformer Ritio R almost maximum [2].

According to the algorithm given above, we construct a bunch train of 4 electron bunches of charge magnitude $Q_0 = 1$ C, $Q_1 = 3$ C, $Q_2 = 5$ C and $Q_3 = 7$ C by using the wakefield produced in the designed structure by one electron bunch. l is the wavelength of AWA (Argonne National Laboratory) which operates at 1300 MHz. The separation between first bunch and second $d_1 = l = 230.610$ mm $= 10.481 \times 22.003$ mm ~ ($10 + 0.5$)l_0. To get ideal transformer ratio, we have to adjust the operating frequency from 13625 MHz to 13625×1.004 = 13679.5 MHz, the separation between first bunch and third $d_2 = (2 + 0.001) l$ and the separation between first bunch and third $d_3 = (3 + 0.003) l$. Or, $d_1 = 230.610$ mm, $d_2 = 461.243$ mm and $d_3 = 692.522$ mm. The adjustments to theoretical results are, for operating frequency $?f = 54.5$ MHz, for separation between the latter bunch and the first bunch $?d_1 = 0$ mm, $?d_2 = 0.001 l = 0.231$mm and $?d_3 = 0.003 l = 0.692$ mm. These results are shown in Table.2 for convenience.

Table.2 The electric parameters and their adjustments in a 4 bunches ramped train for a Transformer Ratio Enhancement experiment

Operating Frequency f	13679.5 MHz
1st Charge Magnitude in RBT Q_0	1 C
2nd Charge Magnitude in RBT Q_1	3 C
3rd Charge magnitude in RBT Q_2	5 C
4th Charge magnitude in RBT Q_3	7 C
The Separation Between the 1st bunch and 2nd d_1	230.610 mm
The Separation Between the 1st bunch and 3rd d_2	461.243 mm
The Separation Between the 1st bunch and 4th d_3	692.522 mm
The Adjustment to the Operating Frequency $?f$	54.5 MHz
The Adjustment to the Separation Between the 1st bunch and 2nd $?d_1$	0 mm
The Adjustment to the Separation Between the 1st bunch and 3rd $?d_2$	0.231 mm
The Adjustment to the Separation Between the 1st bunch and 4th $?d_3$	0.692 mm

By using above prediction, we can get the Transformer Ratio R = 25/3.8 = 6.6, a little less than the ideal result in [2], shown in Fig.4.
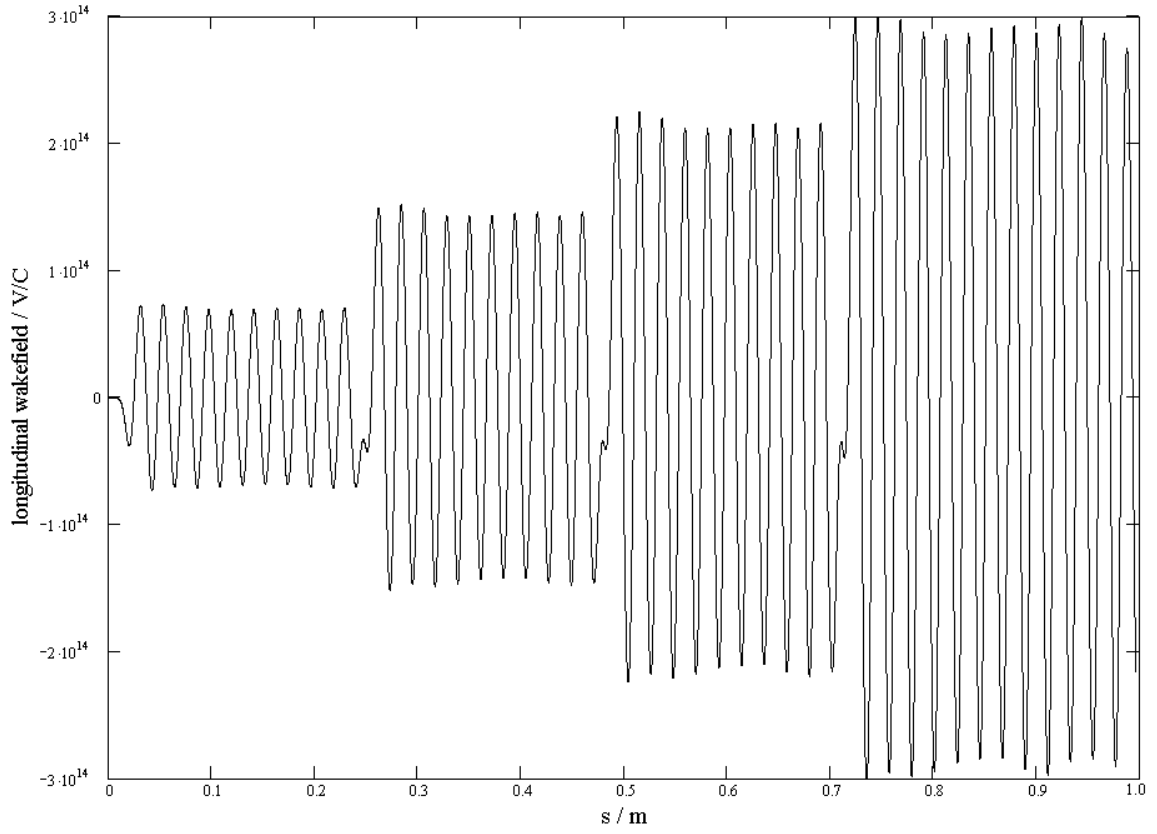

Fig.4 The resultant wakefield produced in the designed structure by a train of 4 electron bunches of bunch length 4 mm and charge magnitude Q_0 = 1 C, Q_1 = 3 C, Q_2 = 5 C and Q_3 = 7 C, the latter bunch separations with first one $d_1 = l$, $d_2 = 2.001l$, $d_3 = 3.003l$, where l = 230.61 mm is the wavelength of AWA. The operating frequency is 13679.5 MHz. The Transformer Ratio R = 25/3.8 = 6.6.

Tolerance Analyzing

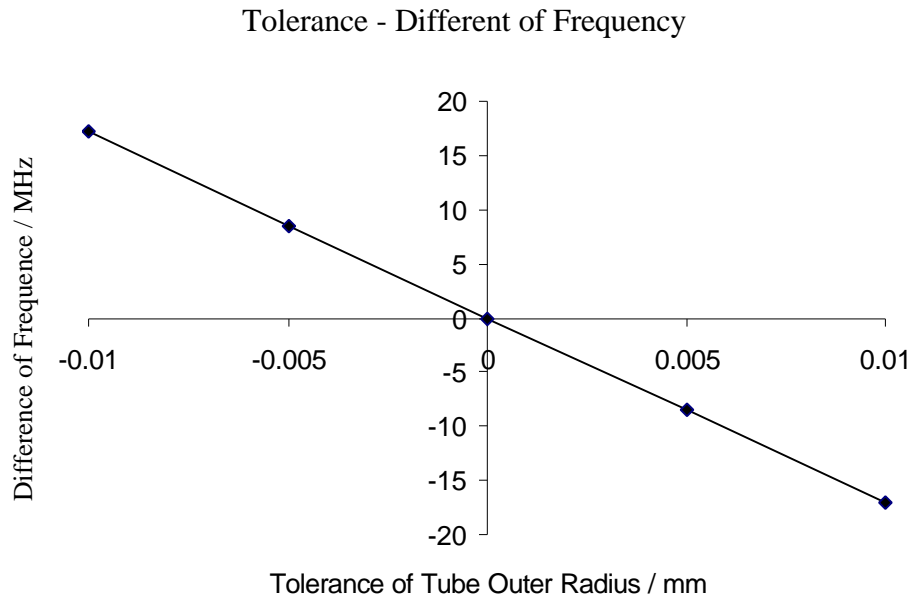For manufacture the above structure, we analyze its tolerance.

Tolerance - Different of Frequency

Fig.5 The difference of frequency vs. the difference of tube outer radius *b*.
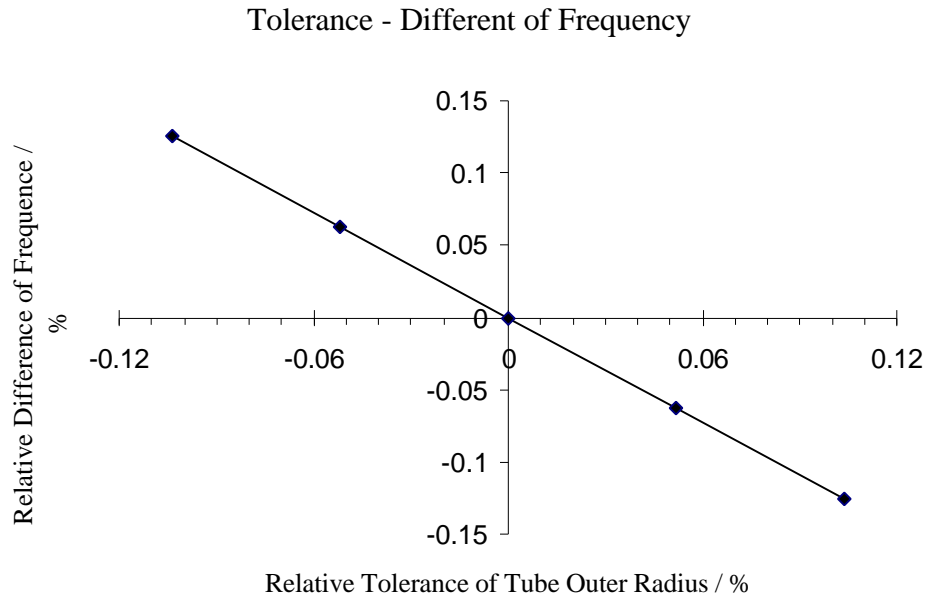
Tolerance - Different of Frequency

Fig.6 The relative difference of frequency vs. the relative difference of tube outer radius *b*.

   The accurate is 5 μm. From Fig.5 and Fig.6, the tolerance 5 μm will introduce about 8.5 MHz frequency shift, or 0.06% relative frequency shift. It is good enough. So we choose tolerance as 5 μm.

Geometry

According to the above results, we have designed the structure shown as Fig. 7 and will manufacture it.
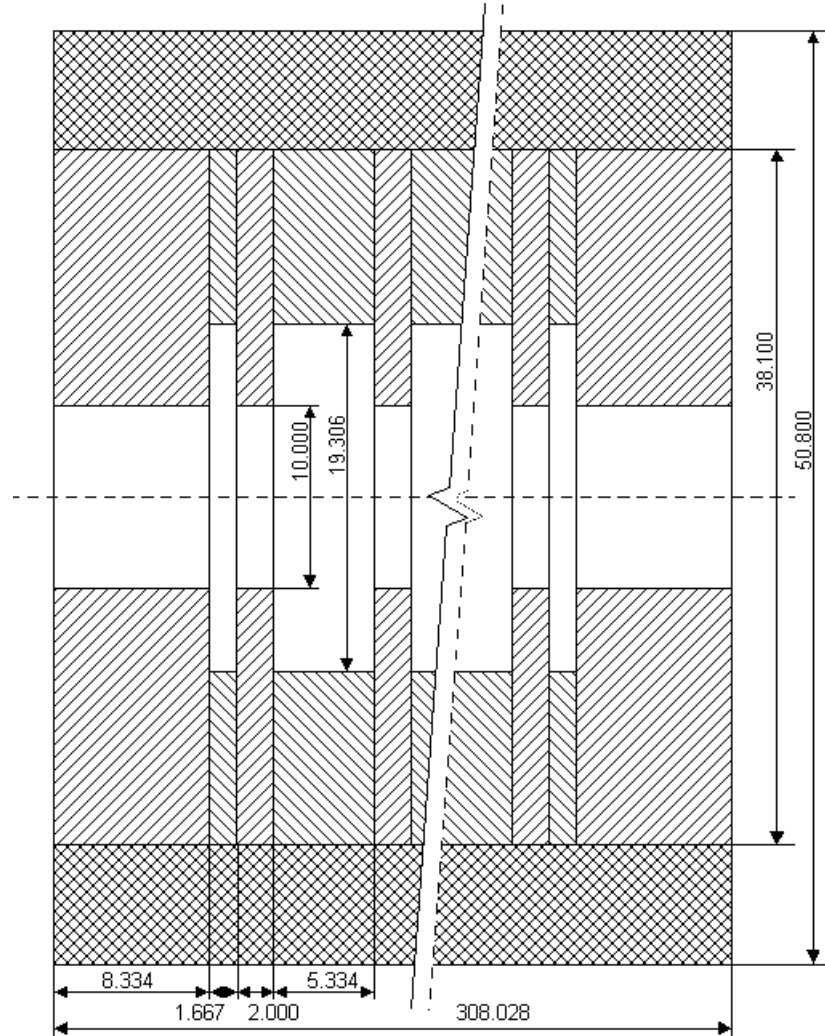

Fig.7 The blue print of the disk-loaded structure for Transformer Ratio Enhancement using RBT in AWA.

Simulation Results of Transformer Ratio Enhancement Experiment

From Fig.4, we can know the simulation results of the planed Transformer Ratio Enhancement Experiment, shown in Table3 and Fig.4.

Table.4   The simulation results of the planed Transformer Ratio Enhancement Experiment

	Peak Decelerating	Peak Accelerating	Transformer Ratio

	Field behind n^{th} Bunch W_n^-	Field behind n^{th} Bunch W_n^+	$R_n = |W_n^+/W_n^-|$
1st Bunch, $Q_0 = 1$ nC	-37.5 kV	71.3 kV	1.9
2nd Bunch, $Q_1 = 3$ nC	-42.8 kV	151.0 kV	3.5
3rd Bunch, $Q_2 = 5$ nC	-42.0 kV	220.3 kV	5.2
4th Bunch, $Q_3 = 7$ nC	-47.3 kV	310.0 kV	6.6

We will measure the energy of the bunches to make sure weather these prediction are right.

Summary

From [2], we know the RBT method to achieve a transformer ratio (R) greater than 2 in any collinear wake field accelerators and the algorithm for designing a RBT experiment. Thus an experiment is planed to be performed at the AWA facility to measure an $R \gg 2$. We designed a disk-loaded structure for this transformer ratio enhancement experiment according to the algorithm of RBT experiment. We designed the geometry of the structure which operates at 13679.5 MHz by using SuperFish and calculated the wake fields in the structure by using Mafia. Then we design a train of 4 drive bunches of bunch length 4 mm and charge magnitude $Q_0 = 1$ C, $Q_1 = 3$ C, $Q_2 = 5$ C and $Q_3 = 7$ C. The separations between the latter bunches and the first one are $d_1 = ?$, $d_2 = 2.001?$ and $d_3 = 3.003?$, where $l = 230.61$ mm is the wavelength of AWA facility. The Transformer Ratio in this experiment is expected to be 1.9, 3.5, 5.2 and 6.6 for 4 bunches separately. We also analyzed the manufacture tolerance.

REFERENCES

[1] P.B. Wilson, Proc. of the 13 th SLAC Summer Inst. on Particle Physics, SLAC Report No. 296, p. 273, E.
[2] J.G. Power, W. Gai, A. Kanareykin, the Advanced Accelerator Concept Workshop at Santa Fe, NM (June, 2000), http://gate.hep.anl.gov/awa/